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Allocation of Clearance Assets in IED Warfare

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Abstract: This article deals with the optimal allocation of mine clearance resources to a network of roads that is continually being mined by an opponent. The motivating application is IED warfare as practiced in Iraq and Afghanistan. The situation differs from most mine warfare analyses in being protracted indefinitely in time, with the processes of mine placement by Red and mine clearance by Blue being intermixed. Each incident involving a Blue convoy and a Red mine removes the mine and possibly causes some damage to the convoy. Blue's object is to minimize the total damage rate by optimally deploying mine clearance forces to road segments. Naval Research Logistics 58: 180–187, 2011

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1. INTRODUCTION

When military forces of one country (or coalition of countries, hereafter referred to simply as "Blue") occupy another country (Green) over a long period of time, it will usually be necessary for Blue to make regular logistic and tactical use of Green's road network. If Green willingly or unwillingly hosts forces (Red) who are opposed to Blue's occupation, then Blue's necessity represents an opportunity for Red, as roads can be mined. In this article, a "mine" is simply a stationary explosive device that Red can secretly place on, under, or near a road, to be detonated at some later time when Blue traffic attempts passage. Blue is assumed to have forces that attempt to clear these mines, the optimal use of which is the subject of this article.

The situation described above fits the occupations of Iraq and Afghanistan by coalitions led by the United States. In each of those countries, the mines employed by Red are often improvised from artillery shells or other kinds of munitions. The resulting mines are usually referred to as Improvised Explosive Devices, or IEDs [see [1], for a review of the effectiveness of these devices in Iraq]. The ready availability of munitions in those countries makes this type of warfare particularly attractive for Red, but the opportunity would be there even without them. Land mines are cheap, effective, and easily transported, so a mine-based attack on Green's road network can be expected in any prolonged, opposed occupation. This analysis is meant to be general enough to encompass all such situations.

In a similar situation, DeGregory [3] employs 0/1 integer programming to optimize allocation of Blue's protective resources. The current analysis results in a different objective function and a nonlinear optimization problem for Blue.

The unit of Blue traffic is taken to be a "convoy," a group of vehicles moving in a coordinated fashion. We will use this terminology even for Blue clearance forces, which are typically Route Clearance Teams (RCTs) employing special purpose vehicles and tools. There are no Red convoys—the term applies only to Blue.

There are multiple reasons for forming traffic into convoys, an important one being economy in providing vehicles such as gun-trucks that protect the convoy from threats other than mines, notably small arms fire. The reason that most directly affects our modeling is the possible provision of convoy "escorts" whose function is to directly precede the rest of the convoy, finding and removing Red mines before they can be effectively detonated. This possibility affects our modeling because the convoy's escorts might be composed of forces that could otherwise be performing general purpose road clearance, which we call "clearance forces" to distinguish them from escorts. Clearance forces pursue their object without reference to the schedule of any particular convoy. It is the optimal employment of clearance forces that is the object of the model introduced latter. Escorted convoys are simply one particular type of Blue traffic, and the effect of the escorts will be reflected in the parameters that characterize the type.

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2. DEVELOPMENT OF THE MODEL

We make four basic assumptions about the Blue versus Red battle on Green's roads:

1. *Indefiniteness:* The battle is assumed to proceed indefinitely, with no time limit. Every mine placed on a road will eventually be involved in an incident with some kind of Blue traffic. This would not be the case if there were no Blue traffic at all, in which case mines would accumulate pointlessly on the roads, but such scenarios are of no interest.
2. *Logistic Ineffectiveness:* Red's efforts are assumed to have a negligible effect on Blue's logistic operations. Although the damage done to Blue convoys may be a significant issue for Blue, the convoys perform their logistic function regardless of damage. When attacked by a mine, a Blue convoy simply continues with its mission, and may even be attacked on multiple occasions. The analytical effect of this is that, in quantifying the probability that a particular mine damages a particular Blue convoy, we will pay no attention to the possibility that other mines might have already damaged the convoy.
3. *Independence:* The various types of Blue traffic and the Red process of placing mines on roads are all assumed to be time-homogeneous Poisson processes, with all Red processes being independent of all Blue processes, and with all processes of any color being independent of each other on any given segment. The significant part of this assumption is Red–Blue independence. We assume that Red does *not* employ tactics such as watching a road segment until a RCT passes, and then rushing out to place a mine in its wake. Red is assumed to be ignorant of Blue's traffic schedules, except for the overall level of the various kinds of Blue traffic (including RCTs). The verity of the Red–Blue independence assumption is arguable and perhaps significant, but we do not intend to argue the point in this article. The time-homogeneous part of the assumption permits the derivation of some simple formulas below, and is probably less arguable — the levels of both Red and Blue processes have fluctuated in Iraq, but with a time constant (months) that is long compared with the decision cycle of either side.
4. *Scalar damage:* We assume that all types of damage due to mines (vehicles lost, vehicles damaged, cargo lost, men killed, men wounded, and so on) can be put on one scale called "damage." Blue's goal is to minimize it.

For generality, we acknowledge several types of mines, indexed by j, as well as several types of Blue traffic, indexed by i. One type of traffic might be RCTs, whereas another might be logistic traffic, and yet another might be troop convoys. The mine types might include buried bombs and roadside bombs, as in Iraq.

We will eventually also need an index k for the road segment, but first consider only one road segment. As the battle is imagined to be indefinite in time, most data is in terms of rates per unit time. For the subject road segment, let

x_j = rate at which Red places mines of type j on the segment

y_i = rate at which Blue traffic of type i uses the segment, in units of convoy passages over the segment per unit time

The same letters in bold type will stand for collections of all such quantities. In accordance with assumption 3, the components of $\mathbf{x}$ and $\mathbf{y}$ are assumed to be the rate parameters of independent Poisson processes. Blue controls $\mathbf{y}$ with the object of minimizing the total damage rate.

The various types of Blue traffic differ quantitatively, rather than qualitatively. Like clearance convoys, logistic convoys can remove mines, and like logistic convoys, clearance convoys are subject to damage. We define the word "incident" to correspond to the removal of an emplaced mine, without regard to whether the causative agent suffers damage. If the mine is of type j and the removal is caused by a convoy of type i, then we will say that the incident is of type (i, j). Some incidents result in damage to Blue and some do not. Let the (i, j) incident probability be

β_{ij} = probability that the passage of one type i convoy results in the removal of one given mine of type j, with or without damage to the type i convoy.

We emphasize that the definition applies to every individual mine of type j that is currently placed on the road segment, rather than to the collection of such mines as a whole. The "given mine" in the definition of β_{ij} might be thought of as a test mine of type j that is placed on the road segment to see how long it remains there before removal, and what the cause of removal is. The placement of such a mine initiates a continuous time Markov process in which the various types of blue traffic all contribute to its removal. The rate of removal by traffic of type i is $y_i\beta_{ij}$, and the total rate of removal is $\mu_j \equiv \sum_i y_i\beta_{ij}$. As this removal rate applies simultaneously to every mine of type j that is still alive (emplaced but not yet removed) on the segment, the number of type j mines still alive is an M/M/∞ queue with input rate x_j and service rate μ_j. The expected value of the number of mines of type j still alive is therefore x_j/μ_j. When a mine of type j is removed, the probability that the removal is by type i traffic is given by $y_i\beta_{ij}/\mu_j$, the ratio of the removal rate by type i traffic to the total removal rate. All of these results are consequences of general theorems pertaining to independent Poisson processes and continuous time Markov chains [5, 9].

When an incident happens, the damage done to Blue is assumed to depend on the mine type and the traffic type:

d_{ij} = expected value of the damage to Blue in an (i, j) incident

As every mine will eventually be involved in an incident of some kind, the total expected rate of damage suffered by Blue on the subject road segment is a summation of terms, each of which is the product of three factors: the mine placement rate x_j, the probability that a mine of type j is removed in an incident of type (i, j) as developed in the previous paragraph, and the damage d_{ij} caused by such a removal. The total damage rate is thus

$$z = \sum_{i,j} x_j d_{ij} (y_i \beta_{ij} / \mu_j). \tag{1}$$

As x_j/μ_j is the expected value of the number of type j mines still alive in the aforementioned M/M/∞ queue, Eq. (1) can also be seen to be the sum of (number of mines) × (incident rate per mine) × (damage per incident), to the same effect.

If mines are subject to removal by activities unrelated to Blue's use of Green's roads, the effect can be handled by introducing a background type of traffic ($i = 0$, say) for which the product $y_0\beta_{0j}$ represents the background rate of removal of each mine of type j. Only the product of the two factors is significant, as Blue will not be in control of y_0. The effect is to give each mine of type j a lifetime of $1/(y_0\beta_{0j})$, even in the absence of other Blue traffic.

Because of assumption 2, extending the analysis to multiple road segments is simply a matter of adding an additional subscript k to all of the parameters and variables. Thus we define $\mu_{jk} \equiv \sum_i y_{ik}\beta_{ijk}$ to be the clearance rate of type j mines on segment k, etc. The final grand total damage rate on all segments is (writing out the expression for μ_{jk}, and using a surrogate subscript l for i in the inner sum)

$$z = \sum_{i,j,k} x_{jk} d_{ijk} \frac{y_{ik}\beta_{ijk}}{\sum_l y_{lk}\beta_{ljk}}. \tag{2}$$

It is Blue's object to minimize this quantity, subject to whatever constraints on $\mathbf{y}$ apply. Equation (2) is true because the expected value of a sum is the sum of expected values, whether or not the random variables being summed are independent. The verity of (2) does not require independence between Blue traffic of type i on segment k and blue traffic of type i on neighboring segments (that assumption is unlikely to be true, and is carefully excluded from assumption 3).

It is worth emphasizing that Blue's object in assigning mine clearance teams is *not* to maximize the rate of removing mines. Indeed, the rate of removing mines is the same as the rate at which Red places them on roads, as every mine is removed by Blue traffic of one kind or another. The object is instead to adjust $\mathbf{y}$ so that mines are removed by traffic that is unlikely to be damaged in doing so, typically by RCTs.

The ultimate damage rate suffered by Blue depends on many things other than the allocation of clearance teams. Blue may be able to influence $\mathbf{x}$ by attacking the Red process that acquires and deploys mines. He may also be able to reduce β_{ijk} by jamming the radio transmitters that are sometimes used to detonate IEDs. He may also be able to reduce d_{ijk} by armoring vehicles. All of these Blue tactics are potentially important, but they do not obviate the need for clearance teams. Formula (2) could be used to assess the effectiveness of each of them, but our interest here is only in the dependence of z on $\mathbf{x}$ and $\mathbf{y}$.

In our model, z is proportional to $\mathbf{x}$. Should Red manage to double all components of $\mathbf{x}$, then the damage suffered by Blue will also double. Should $\mathbf{x}$ become sufficiently large, then assumption 2 will eventually become untenable, but until that happens, scalar factors multiplying $\mathbf{x}$ are tactically insignificant as far as clearance teams are concerned.

3. MINIMIZING TOTAL DAMAGE

In this section, we consider the problem of minimizing total damage, subject to constraints on $\mathbf{y}$, for a given value of $\mathbf{x}$ (we will consider a game where Red controls $\mathbf{x}$ in Section 4, but $\mathbf{x}$ is simply data in this section). Some components of $\mathbf{y}$ will not be controllable. For such traffic types i, y_{ik} is given for all k. Otherwise, assume for the moment that the only constraint is that the total amount of controllable traffic of type i must not exceed C_i convoys per unit time:

$$\sum_k y_{ik} \le C_i, \quad \text{for all controllable } i. \tag{3}$$

EXAMPLE 1: Suppose there are two types of traffic traversing two road segments and Red uses only one type of mine. Type 1 traffic is controllable RCTs, which are assumed to be invulnerable to mines (so $d_{111} = d_{112} = 0$). Type 2 traffic is vulnerable logistic traffic, which is not controllable (so y_{21} and y_{22} are given traffic levels). We take β_{ijk} to be 1 in all cases. There are only two nonzero terms in (2), and the expression reduces to

$$z = \frac{x_{11}d_{211}}{1 + y_{11}/y_{21}} + \frac{x_{12}d_{212}}{1 + y_{12}/y_{22}}. \tag{4}$$

As y_{21} and y_{22} are not controllable, the only variables in (4) are the two clearance effort levels y_{11} and y_{12}, which should

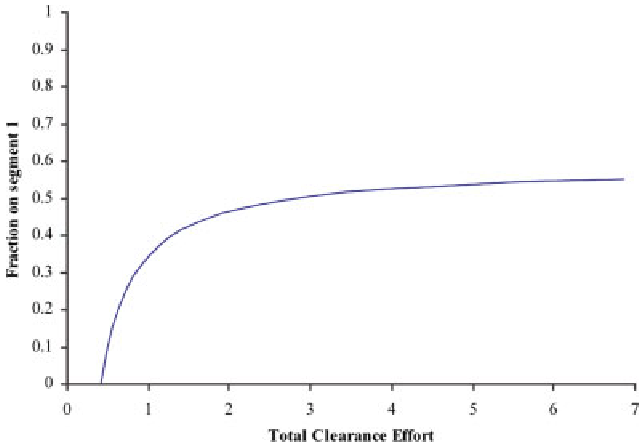

Figure 1. Fraction of effort devoted to the first segment when the logistic traffic levels on two segments are 2 and 1, respectively, versus total clearance effort. Unless the total effort exceeds 0.5, all effort should be devoted to clearing the segment with less logistic traffic.

be selected by Blue to minimize z, subject to (3). The numerator of each term can be thought of as a "default" rate of damage to traffic on that road segment that applies when no clearance effort is allocated. The denominator is a reduction factor that increases linearly with effort.

The default rate of damage does not depend on the logistic traffic level—if there were no clearance effort, damage to Blue would depend entirely on Red's ability to place mines on roads. The logistic traffic level instead affects the efficiency of clearance, with high traffic levels making clearance difficult. Because of this feature, given two road segments that are identical except for logistic traffic levels, and given only a small amount of clearance effort, that small amount should be devoted entirely to the segment with the least logistic traffic, a conclusion that may be counterintuitive.

Figure 1 shows the fraction of total effort that should be devoted to segment 1 when $x_{11} = x_{12} = 1$, $d_{211} = d_{212} = 1$, $y_{21} = 2$ and $y_{22} = 1$. If the total effort is 0.5, segment 2 should get all of the effort and the total damage rate will be $1 + 2/3 = 1.67$. One might expect the optimal clearance effort on each segment to be proportional to the logistic traffic level. If clearance effort was allocated using that principle, then we would maintain the ratio $y_{11}/y_{12} = 2$, regardless of total clearance effort, and the curve in Fig. 1 would be flat at a height of 2/3. For a total effort of 0.5, the total damage rate would be $z = 6/7 + 6/7 = 1.71$, as can be seen by substituting $y_{11} = 1/3$ and $y_{12} = 1/6$ into (4). Although 1.71 is not that much larger than 1.67, this example still demonstrates that the proportionality principle is not a reliable guide to effort allocation. There are two competing effects going on. One is diminishing returns as the allocation of effort to a given segment is increased and the other is "competition" from logistic traffic. When the total clearance effort is small, the second effect is dominant and all effort goes to the segment where competition is least.

Another way of resolving the counterintuitive nature of Example 1 is to recall the naval maxim that "every ship gets to be a minesweeper once." In this sense, heavy logistic traffic does its own clearance.

The objective function (2) is a convex function of $\mathbf{y}$ in Example 1, but this is not true in general. If the objective function is not convex, there may be local optima that are not global. The next two examples illustrate when such difficulties can be expected.

EXAMPLE 2: Reconsider Example 1, except make $y_{21} = y_{22} = 1$; that is, logistic traffic is at unit level on both segments. Also make $x_{11} = x_{12} = 1$ and $d_{211} = d_{212} = 1$, so both segments have identical parameters. The "RCT" curve in Fig. 2 shows total damage z as a function of y_{11} when there are two units of clearance effort in total and $y_{12} = 2 - y_{11}$. This curve is a convex function with a unique minimum at $y_{11} = 1$. The optimal clearance plan treats both segments equally. This is not surprising, since both segments have identical parameters.

EXAMPLE 3: The same as Example 2, except that RCT effort is fixed at 1 on each segment, while logistic traffic is controllable, subject to totaling 2 convoys per unit time. The problem is now to allocate logistic traffic to the two segments (imagine a source connected to a sink by two parallel segments), rather than RCT effort. Total damage as a function of the logistic traffic on the first segment is shown as the concave curve in Fig. 2. Even though the two segments are identical, the optimal solution puts all logistic traffic on one segment, rather than split between the two, with the only local extremum being a maximum, rather than a minimum.

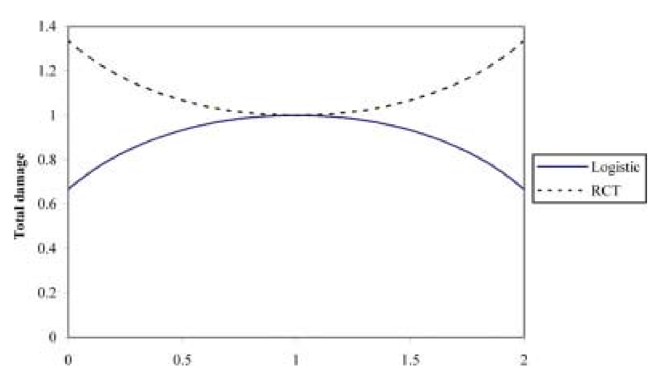

Figure 2. The RCT curve shows total damage when clearance effort is controllable, subject to totaling 2, and there is unit logistic traffic on each of two segments. The Logistic curve shows total damage when logistic traffic is controllable, but clearance effort is not.

Example 3 is an implausible model of reality, given our expectation that route clearance effort will be controllable rather than logistic traffic. However, it does serve to make the point that z in Eq. (2) is not necessarily a convex function of the controllable part of **y**, and that local optima may not be global. In problems where the controllable part of **y** represents traffic that is itself subject to damage from mines, as is often true in the dangerous business of mine clearance, there may be multiple local extrema, some of which are actually maxima, with the global minimum possibly being on a boundary. All of these difficulties occur in Example 3, but none of them occur when controllable traffic is invulnerable, as in Examples 1 and 2. When there are no controllable variables in the numerators of (4), the objective function is convex, and all local optima are global [2].

All of the above examples employ the simple constraint (3) on **y**. More generally, we might require that **y** be an element of some given convex set C, rather than simply constrain the total amount of effort. This might be useful if controllable traffic had a home base(s) located in a network, so that travel over certain segments forces travel over others. See Washburn and Wood [12] for an example of such a formulation. The resulting minimization problem is well suited to optimization packages such as Excel's Solver ([6], see Appendix), or in larger problems to more capable packages such as GAMS [8]. As long as every controllable component of **y** represents an invulnerable type of Blue traffic, all local optima are global.

4. AN ALTERNATIVE VIEW OF DAMAGE MINIMIZATION

In the minimization problem considered in Section 3, **x** is a known quantity when Blue chooses **y**. The usual context for this view is that **x** can be observed repeatedly in successive periods, with **y** being adjusted in each period to be optimal against the most recently observed **x**. There is a long history of the use of this point of view in analyzing warfare [7], and we intend it to be the principal one here. One of its main advantages is that the logistic situation faced by Red in placing mines on roads does not have to be understood by Blue — it suffices to observe the results of Red's efforts. Nonetheless, if Red has an information advantage over Blue, there is something to be said for the point of view that Red will in effect know **y** when determining **x**, and will choose **x** to maximize total damage. This alternative viewpoint is explored in this section.

It is still Blue's object to choose **y** to minimize total damage, but now he must do so in the knowledge that Red, once he knows **y**, will respond with the worst possible value for **x**. To analyze this situation, we first introduce the quantity

$$a_{jk}(\mathbf{y}) = \frac{\sum_i d_{ijk} y_{ik} \beta_{ijk}}{\sum_i y_{ik} \beta_{ijk}} \tag{5}$$

With that definition, Eq. (2) can be written

$$z = \sum_{j,k} x_{jk} a_{jk}(\mathbf{y}). \tag{6}$$

This suits our purpose because all of the dependence on **y** is incorporated in $a_{jk}(\mathbf{y})$, and $a_{jk}(\mathbf{y})$ does not depend on **x**.

As Red chooses **x** to maximize total damage, we need to make an assumption about the constraints that Red faces in doing so. We assume a simple constraint where each mine of type j placed on road segment k costs c_{jk}, and Red has a certain budget available for placing mines of all types. Within this cost structure, Red's best option is to choose a mine type and segment for which the ratio $a_{jk}(\mathbf{y})/c_{jk}$ is maximized, the maximum bang-per-buck. This is true regardless of Red's budget limitation, as long as it is small enough to preserve the plausibility of the assumption regarding logistic ineffectiveness, so Blue does not need to know Red's budget in choosing **y**. Blue's problem is to make sure that the maximum bang-per-buck ratio (call it r) is as small as possible. Thus, Blue's problem is to

$$\text{minimize } r, \text{ subject to } a_{jk}(\mathbf{y}) \le r c_{jk} \quad \text{for all } j, k, \text{ and } \mathbf{y} \in C. \tag{7}$$

where C is the set that constrains **y**, possibly (3). The variables in (7) are r and **y**. The constraints are to the effect that the bang-per-buck ratio cannot exceed r for any (j, k), thus assuring that r has the desired meaning.

Solutions of (7) sometimes involve large amounts of clearance effort being spent on segments where Blue is not very effective at clearance—even if $a_{jk}(\mathbf{y})$ is difficult to make small, (7) requires that it *must* be made smaller than rc_{jk}. Solutions of (7) will therefore sometimes differ significantly from solutions of (2). A specific example will be considered in Example 4 below, and Washburn [11] includes additional examples.

The mathematical program (7) is generally nonlinear because of the ratio that defines $a_{jk}(\mathbf{y})$ in (5). If the numerator of (5) were a constant, however, as it would be if only uncontrollable traffic were vulnerable to mines, then (7) could be reduced to a linear program where the reciprocal of r is maximized. Again, we see that problems where controllable traffic is invulnerable to mines are essentially simpler than problems in general.

One could also imagine a two-person zero-sum game where the payoff function is (6), with player 1 (Red) controlling **x** and player 2 (Blue) controlling **y**. As (6) is a concave (in fact linear) function of **x**, the value of that game is the same as its max min value [10]. However, there is little to recommend this game theoretic point of view because Blue's minimizing strategy will in general be mixed, and it is hard to imagine how Blue could maintain the secrecy of his strategy over a long period of time, or even maintain the meaning of **y** as a collection of Poisson process rates. The more defensible, larger min max value is determined by (7), except that the optimal r needs to be multiplied by Red's budget to find the damage ultimately suffered by Blue. The minimizing **y** does not need to be a secret in the min max problem, and its constancy in time permits the usual Poisson interpretation.

5. PARAMETER ESTIMATION

The parameters used in specifying the model of Section 2 will not usually be directly observable by Blue. In this section, we show how they can be estimated using measurements associated with current operations. It is easiest to imagine that all data are gathered over some initial test period of length T during which traffic levels of all types remain constant. Suppose we measure, for all i, j, and k,

B_{ijk} = number of incidents of type (i, j) on segment k over the period
D_{ijk} = total damage from incidents of type (i, j) on segment k over the period
Y_{ik} = level of traffic of type i on segment k over the period

The sum $\sum_i B_{ijk}$ is the total observed number of type j mines removed from segment k over the time period. The expected number of mines placed on the segment according to the model is $x_{jk}T$. If T is long enough that the two can safely be equated, we have

$$x_{jk} = \left(\sum_i B_{ijk}\right)/T. \tag{8}$$

This is our estimate of the rate at which mines are placed on the segment. Similarly, an estimate of d_{ijk} is

$$d_{ijk} = D_{ijk}/B_{ijk}, \tag{9}$$

the average damage per incident of type (i, j) on segment k. Finally, let $M_{jk} = \sum_i \beta_{ijk} Y_{ik}$. The expected number of (i, j) incidents on segment k over a period of length T, according the model, is $x_{jk}T\frac{\beta_{ijk}Y_{ik}}{M_{jk}}$, which should be roughly equal to the observed number B_{ijk}. Equating the two, we can estimate β_{ijk}:

$$\beta_{ijk} = \frac{B_{ijk}M_{jk}}{x_{jk}TY_{ik}} \tag{10}$$

Using Eqs. (8), (9), and (10), we can now write the formula for zT in terms of observable quantities:

$$zT = \sum_{i,j,k} x_{jk}\frac{D_{ijk}}{B_{ijk}}\frac{\frac{B_{ijk}M_{jk}y_{ik}}{x_{jk}TY_{ik}}}{\sum_l \frac{B_{ljk}M_{jk}y_{lk}}{x_{jk}TY_{lk}}} = \sum_{i,j,k}\left(\sum_l B_{ljk}\right)\frac{D_{ijk}\frac{y_{ik}}{Y_{ik}}}{\sum_l B_{ljk}\frac{y_{lk}}{Y_{lk}}} \tag{11}$$

The second equality in Eq. (11) is obtained by substituting Eq. (8) for x_{jk} and cancelling common factors. The result is a formula for the damage to be expected over a time period of length T, as a function of observable parameters and the as yet undetermined variables **y**.

The length of the observation period T does not appear on the right-hand-side of Eq. (11), and is therefore irrelevant to the optimization of **y**. However, there may be statistical issues if some of the observed data consists of small integers. This is most likely to be the case in estimating d_{ijk}, the damage per incident of type (i, j) on segment k, as the numerator D_{ijk} in Eq. (9) might very well be 0 on some road segments. If there is no reason to suspect damage per incident to depend on the road segment, one might replace Eq. (9) with

$$d_{ij} = \left(\sum_k D_{ijk}\right)/\left(\sum_k B_{ijk}\right), \tag{12}$$

and then derive an expression for zT on the basis that every road segment has a damage parameter given by Eq. (12). The result is

$$zT = \sum_{i,j,k}\left(\sum_l B_{ljk}\right)d_{ij}\frac{B_{ijk}\frac{y_{ik}}{Y_{ik}}}{\sum_l B_{l_{jk}}\frac{y_{lk}}{Y_{lk}}}, \tag{13}$$

where d_{ij} is given by Eq. (12). Other kinds of aggregation might also be called for when T is small, or consider Bayesian techniques [4].

Two points are worth noting about the equation for zT, whether Eqs. (11) or (13). The first is that, if index i corresponds to an uncontrollable type of Blue traffic, then Y_{ik} does not need to be known because y_{ik} will necessarily be the same as Y_{ik}, and only the ratio is needed. This is important because the levels of uncontrollable Blue traffic may very well be unknown. The second point is that, as long as no damage has been done to any kind of controllable traffic over the test period, Eqs. (11) and (13) are both convex functions of the controllable part of **y**. As long as **y** is constrained to be in any convex set, all local minima will therefore be global.

The implied use of measurements such as these is that measured data, including **x**, is assumed to be constant over some period in the future, even if **y** changes. An alternative point of view is that **x** will be chosen as an optimal countermeasure to **y**, as described in Section 4. In that case Eq. (8) must be replaced by the constraints that Red faces in determining **x**.

EXAMPLE 4: As in Example 1, there is only one kind of mine and two kinds of blue traffic: vulnerable logistic traffic is Type 2 and invulnerable RCT traffic is Type 1. However, the data shown in the first four rows of Table 1 is realistic data collected in Iraq for eight road segments during one 17-week period in 2007-8. The mines are all IEDs, and every attack by an IED is classified as either "effective" or "ineffective." The

Table 1. Analysis of an eight-segment road network in Iraq over a 17-week period.

	Segments (k)								
	1	2	3	4	5	6	7	8	Totals
D_{21k}	8	10	9	4	4	2	8	5	50
B_{21k}	31	38	42	41	32	10	19	41	254
B_{11k}	73	58	51	41	39	18	34	76	390
Y_k	965	819	983	908	848	87	37	85	4,732
y_k	1,085	1,123	1,001	307	425	186	286	319	4,732
D_{21k} (revised)	7.36	8.17	8.91	5.98	5.51	1.15	1.51	1.79	40.38

first row of data counts the number of effective attacks on logistic traffic for each segment. The second row is the number of incidents involving logistic traffic and the third row is the number of incidents involving RCTs (every incident by definition removes exactly one IED). The total effort in Row 4 is 4732 RCT-hours. The last two rows show the forecast results if the same total effort is optimally distributed over the segments in a subsequent 17-week period where the segments see the same number of IEDs, using Eq. (11) for parameter estimation. Roughly speaking, the optimal allocation takes RCT-hours away from Segments 4 and 5 to increase the allocations to 6, 7, and 8. The overall result is to decrease the number of effective attacks from 50 to 40.38, a decrease of about 19% (this result, as well as others in this section, were obtained using the spreadsheet described in the Appendix).

If Eq. (13) is used for parameter estimation instead of Eq. (11), we observe a smaller redistribution of RCT-hours and a smaller decrease to 43.46 effective attacks over the 17-week period. Formula Eq. (9) has the fraction of attacks that are effective on Segment 7 being 8/19, the largest of all the segments, a fact that is exploited when Eq. (11) is used to allocate RCT hours. When Eq. (9) is replaced by Eq. (12), all of the segments have the same fraction of effective attacks, and the benefits of redistribution are reduced.

During the 17-week period, there were eight effective attacks out of 53 IEDs placed on segment 7, so the number of effective attacks per IED on that segment (call it the segment's "productivity") is $8/53 = 0.15$. From Red's viewpoint, Segment 7 is the most productive segment. If all $254 + 390 = 644$ IEDs used over the period had been placed on that segment, there would have been $644 \times 8/53 = 97.21$ effective attacks, according to our model, rather than only 50. To guard against such disasters in the future, Blue might choose $\mathbf{y}$ to minimize the largest productivity among the eight segments. This is equivalent to solving Eq. (7) with the cost of an IED being the same on all segments. In this example, minimizing the largest productivity leads to all segments having the same productivity, namely, 0.0755. The worst case damage is then $0.0755 \times 644 = 48.65$, considerably smaller than 97.21. Unfortunately, the damage is still 48.65 even if Red's habits do not in fact change in the future, whereas it can be reduced to 40.38 if Red's lethargy can be successfully predicted. The allocations resulting from the two different points of view can be seen to differ significantly.

The potential impact of force level changes can be assessed by changing the number of RCT hours available. If force levels are doubled, for example, so that 9464 hours are available over a 17-week period, and then optimally distributed, the number of effective attacks decreases from 40.38 to 25.61. Such computations might be of use in allocating RCT resources between different areas of operations within Iraq, or even between Iraq and other theaters.

We have two reasons for basing Example 4 on real data. One is to make it clear that RCT allocations made in the absence of an effectiveness model can differ substantially from optimality. The other is to enable one final comment about using our model, or any other model that depends on similar inputs. Incidents that result in the removal of an IED happen in well defined places at well defined times, and are therefore relatively easy to keep track of. This accounts for $D_{21\mathrm{k}}$, $B_{21\mathrm{k}}$, and $B_{11\mathrm{k}}$ in table 1, but not Y_k. Keeping track of Y_k requires a system for recording RCT patrols that is widespread and uniform enough to permit aggregation of effort over road segments and military units. As of this writing (early 2009), such a system does not exist in Iraq. The data in that row of Table 1 were obtained only after a significant investment of man-hours in recalling the history of those eight road segments over one specific period of time, and may very well be erroneous in spite of that effort. Partly because of lack of information about Y_k, no system of allocating RCT hours optimally has ever been employed in Iraq, either within or between theaters, to our knowledge. Organizationally speaking, the first priority in developing a procedure that will optimally and regularly allocate clearance effort over a large region is to institute a procedure for keeping track of past clearance effort.

6. SUMMARY

We have introduced a simple model of mine warfare as practiced over a long time period on a network of roads.

In spite of its fundamental simplicity, the model sometimes leads to counterintuitive conclusions, particularly when the forces that clear mines are themselves vulnerable. In particular, it is not necessarily true that clearance effort should be proportional to logistic traffic levels.

Much depends on the extent to which Red's behavior frustrates Blue's attempts to minimize the total damage rate. We have investigated two extreme cases: Section 3 covers the case where Red's behavior is stable in the face of Blue's adjustments of clearance effort, whereas Section 4 covers the case where Red's behavior is the worst possible reaction to Blue's adjustments. The truth is no doubt somewhere in between. Given the choice of the two extremes, we favor the first because it does not require assumptions about the constraints faced by Red in finding the worst possible reaction.

No model will be of any use unless the data required for estimating its parameters are available. Estimation of data based on observable quantities is covered in Section 5. The quantities that must be observed include Y_k, the amount of clearance effort applied to road segment k over some time period. Any system such as ours for optimally allocating clearance effort on a regular basis must include a provision for keeping track of Y_k, in addition to incident-based data.

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APPENDIX

A Microsoft Excel [6] workbook named *MineRoad1.xls* that implements some of the calculations discussed earlier can be downloaded from the "Downloads" link at http://faculty.nps.edu/awashburn/. It has five sheets, the first three of which deal with Example 4. The optimization engine is Excel's Solver.

- Sheet "Iraq(1)" implements (11) in Example 4.
- Sheet "Iraq(2) implements (13) in Example 4."
- Sheet "Iraq(3)" implements (7), a game theory analysis of Example 4 with $c_{jk} = 1$ for all (j, k).
- Sheet "2by3by4" implements (2) for the case where there are two traffic types, three mine types, and four segments. Both traffic types can be vulnerable, depending on inputs. As distributed, Solver has found a local optimum that is not global — a better $\mathbf{y}$ is (0,0,4,0).
- Sheet "Figure 1" is the source of Fig. 1.
- Sheet "Figure 2" is the source of Fig. 2.

The workbook is a suitable way for the reader to experiment with his own variants of Examples 1–4, or with problems entirely of his own invention.

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